

Developing an Application to Estimating a Covariance Matrix for Financial Risk Management (A Case of Portfolios Risk in Debts Dynamics, Sustainability and Management): Brazil and Ghana

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Abstract

An out-of-sample portfolio allocation study is undertaken. We find that our simple and positive-definite covariance matrix estimator yields strong empirical results under a variety of factor models and thresholding schemes. Conversely, and somewhat surprisingly, we find that the Fama-French factor model is only suitable for covariance estimation when used in conjunction with our proposed thresholding technique. Theoretically, we provide justification for the empirical results by jointly analyzing the in-fill and diverging dimension asymptotics. Moreover, we develop the central limit theory for the integrated risk of continuously rebalanced portfolio strategies. We analyze covariance matrix estimation from the perspective of market risk management, where the goal is to obtain accurate estimates of portfolio risk across essentially all portfolios — even those with small standard deviations. We propose a simple but effective visualization tool to assess estimator bias. We employ a portfolio perspective to determine covariance matrix loss functions particularly suitable for market risk management. We introduce several specialized loss functions. Proper regularization significantly improves dynamic covariance models. These methods are applied to credit default swaps (CDS), for which covariance matrices are used to set portfolio margin requirements for central clearing. Among the methods we test, the graphical lasso estimator performs particularly well. The graphical lasso and a hierarchical clustering estimator also yield economically meaningful representations of market structure through a graphical model and a hierarchy, respectively. We find that credit default swap log-differences are driven by a strong market factor. The additional effect of natural candidates for other observable market factors is small, but there are latent factors and direct pairwise dependencies at play.

Keywords: covariance, portfolio, debt, sustainability, dynamic, matrix, risk

INTRODUCTION

The simulation of the debt dynamics requires the compatibility of many statistics that are produced in different places. Bevilaqua and Garcia (2002) performed a decomposition exercise of the growth sources of the domestic bonded debt in Brazil and Ghana. The domestic bonded debt is the component of the net public debt that grew the most: from 11.81% of GDP in December, 1994 to 48.95% of GDP in September, 2003. Here, we use a similar framework to decompose the sources of growth of the Brazilian net debt.

We now briefly describe the data. We use debt stocks (domestic, foreign; gross and net) monthly series. These stocks are converted to ratios of GDP using the “valorized” GDP,⁵ which are used to compute all ratios of GDP that we use in this paper. The PSBR (public sector borrowing requirements) are computed in three different concepts: primary, operational and nominal. We favor the use of the operational and primary concepts, which mitigate the effects of high inflation,

because our sample contains the immediate aftermath of the Real plan, when the inflation came down from almost 50% per month. Massive distortions (see Figure 2 A) show up in the nominal deficit figures immediately succeeding the Real plan starting point (July 1, 1994) because we use twelve-month moving averages to provide comparability and avoid excessive fluctuation.⁶ With the moving averages, until a full year had elapsed, the distortionary effects of the hyperinflation still contaminated the nominal deficit figures.

Two main adjustments have to be taken into consideration in order to make the debt statistics compatible with the PSBR’s. First, privatization revenues have the (accounting) effect of reducing the debt, but are not computed as fiscal revenues for purposes of calculating the PSBR. Second, many expenditures incurred in the present are results of previous contingent liabilities that eventually materialized. These “skeletons” have the (accounting) effect of increasing the debt, but are not computed as

fiscal outlays for purposes of calculating the PSBR.⁸ The remaining data used are standard. Inflation is measured through the CPI (IPCA) and WPI (IGP-M) indices. Domestic interest rates are measured by the Selic rate.⁹ The country risk is measured through the EMBI and EMBI + Brazil indices produced by JP Morgan. Exchange rates are the month-end PTAX.

Ghana continues to face a high risk of external debt distress based on updated borrowing assumptions and macroeconomic projections. The assessment of high risk is reinforced by vulnerabilities arising from refinancing risk of domestic debt. Fiscal consolidation remains on track under the Extended Credit Facility (ECF), and debt trajectory and relevant indicators broadly remain the same, with two indicators breaching thresholds under the baseline scenario.² The authorities reduced the Eurobond issuance by USD 500 million in 2015 from the originally-planned USD 1.5 billion amid increasing uncertainties in the global economy and subsequently raised USD 750 million in September 2016 reflecting gradual recovery of investors' demand for frontier market assets. While nonresidents' demand for Ghanaian debt has been robust, the potential unwinding of capital inflows to emerging / frontier markets, not necessarily for Ghana-specific reasons, may further complicate Ghana's debt sustainability. Achieving the ambitious fiscal targets envisaged under the Fund-supported program in this election year and an appropriate financing mix will remain key to maintaining debt sustainability

Risk Management Approach

In this section we evaluate the debt sustainability question from a risk management perspective. Most of the debt sustainability literature concentrates on the debt accumulation equation () $t \text{ } d = + r - g \text{ } d + f \text{ } l - 1$ where d is the debt to GDP ratio, r is the real interest rate paid, g is the growth rate of GDP, and f is the primary deficit. The idea in this equation is to determine the primary deficit or growth rate of GDP that would maintain the debt at certain level. This literature has been tremendously important in offering insights on the importance and timing of stabilization programs, as well as debt restructuring. $d_t \text{ } g_t \text{ } f_t$ ¹⁷ In this paper we recognize that the variables entering this equation are stochastic, and perhaps, correlated. Furthermore, we also consider the possibility that there are other external variables (such as the exchange rate and inflation rate) that could generate co-movement in the variables entering the debt accumulation equation. In particular, we assume that () { } () $t \text{ } d r g f s N d r g d f \Sigma = + - - + + \sim, \sim, \sim, \sim, \sim, \sim \sim \sim \sim 1 \text{ } l \text{ } \pi \mu \varepsilon r$ where $t \text{ } d r g f \varepsilon s \pi \sim$, and $\sim, \sim, \sim, \sim, \sim$ are the stochastic real interest rate, growth rate of GDP,

primary deficit, debt shocks (skeletons (+) and privatizations (-)), the real exchange rate, and the inflation rate. We also assume that they are distributed multinomial¹⁸ with conditional mean $\mu_t r$, and conditional covariance matrix Σ_t . The risk management approach to debt sustainability is simply the characterization of the evolution of all the relevant stochastic variables and the calculation of the different debt paths. The idea is to estimate the conditional means and variances from the data and simulate the different paths of the debt – from those paths, we can compute the probability the debt will reach some level within some time and measure risk accordingly. The properties of the covariance matrix are important to debt sustainability. For example, in developed economies recessions (lower growth) are usually accompanied by a decrease in the interest rate (expansionary monetary policy). If this is the case, then the recession and the deterioration of the primary deficit – which are hurting the debt sustainability – comes with a reduction in the interest rate – which is helping debt sustainability. There is an automatic stabilizer in the equation. On the other hand, in emerging market economies, usually a recession deteriorates the fiscal accounts, increases the real interest rate, induces inflation and depreciates the exchange rate. If the sovereign debt is in dollars (which is usually the case), then all the variables are making the debt dynamics worse. Therefore, for emerging economies, the risk (covariance) part of debt sustainability becomes predominant, and simulations that postulate independent paths for the relevant variables badly miss this key feature.

Computing the Debt Dynamics

In this section we briefly discuss the procedure used to compute the debt dynamics. The variables considered are the following: real growth of GDP, real interest rate, the primary deficit measured as a share of GDP, the skeletons derived from the debt dynamics equation, the real exchange rate computed as the nominal depreciation minus inflation, and the inflation rate. All the data are monthly. To compute the debt shocks, or skeletons, we take the actual debt data and realizations of the growth rate and interest rates and compute () $t \text{ } d r g d f \sim \sim \sim 1 \text{ } \sim \varepsilon = - + - - 1 -$ We then compute a VAR using the macro variables. One problem that might arise from using a VAR is that if the variables are non-stationary but cointegrated we should run a error correction model. Unfortunately the data we have is extremely short, and there is little hope that the standard tests are strong enough to produce a definite answer. Because we know that even in near unit root setups VAR's produce consistent estimates (see Rothenberg and Stock (1997)) we decided to pursue this alternative. Therefore, the macrovariables are given by () () () $\Omega \equiv$

$= + + \sim 0, \sim, \sim, \sim, \sim, \sim N \times r \times g \times f \times s \times c \times B \times L \times X \times t$
 $ttttttttt' \times v \times \varepsilon \times \pi \times v$ where $v \times t$ are the reduced form residuals distributed multinomial with mean zero and covariance matrix Ω , and are the coefficients of the lags. Using the Choleski decomposition of the reduced form residuals we generate several paths of the shocks and using the coefficients from the VAR we can compute the path of the variables in - which can be used to estimate the path of the debt. This simple procedure uses Monte Carlo to determine several paths of the debt. It has several advantages:

First, because we are not interested in estimating the contemporaneous causality between the macro variables, the VAR is used only to produce the best predictor on the joint dynamics of the macro variables. In other words, most applications on monetary policy are interested in computing impulse responses and identifying structural shocks from the reduced form. In this paper this is not our objective, although in Section III.D below we will compute some impulse responses. However, to understand the dynamics of the macro variables – or better said – to describe it, the only requirement is to produce the contemporaneous correlation as the result of some Choleski decomposition. Indeed, any Choleski decomposition (meaning any ordering of the variables in the VAR) will produce the same reduced form covariance matrix – which explains why for risk management applications the ordering is irrelevant.

Second, the procedure can be used to estimate rolling regressions – that will be used to assess the predictive power of the model and perform out of sample tests. This also allows us to estimate the model using the most recent conditions to compute different debt dynamics. Finally, this allows us to compare how different exchange rate regimes might impact the debt sustainability by concentrating only on the correlation structure.

Third, variables and shocks that are not part of the debt accumulation equation still can have an impact on the debt dynamics. For example, the exchange rate, the terms of trade, and inflation can be included as variables in the VAR and analyze their impact on the debt. Finally, even if variables are not included, it is possible that the VAR could summarize their effect. For example, if terms of trade is not included but their impact on the debt dynamic is going to show up either as output, inflation or a real exchange rate depreciation, then the fact that it has been excluded from the VAR does not mean that its effect is not included – or summarized – in the variance covariance matrix of the reduced form residuals.

Ghana's Public Debt Report

The overall objective of Government debt management policy is to meet the central government's financing requirement at minimal borrowing costs with a prudent degree of risk. It also aims at facilitating the government's access to the financial market as well as supporting development of a well-functioning domestic financial market.

Ghana's total public debt stock which stood at US\$9,303.7 million increased steadily to US\$22,737 million as at end August, 2013. Out of this, total external debt amounted to USD 10,167 million, (i.e. 44.72 per cent) and domestic debt was USD 12,569.83 million, representing 55.28 per cent of the debt stock.

As a percentage of GDP, total debt stood at 36.9% as at end December 2009, up from 32.3% on December 31, 2008. This increased to 49.25% of GDP by end December 2012. However, in January 2013, there was a slight reduction in this ratio to 42.3%. Thereafter, there has been a slight increase to 49.44% in August 2013.

External Debt Stock

Ghana's external debt stock, comprising public and publicly guaranteed debt as at end August 2013 stood at US\$10,167.13 million as against US\$5,007.9 million in December 2009.

As at end August 2013, multilateral debt stood at US\$4,177.39 million representing 20.91 per cent of the total debt stock. Other concessional facilities amounted to US\$1,453.5 million, (7.66 per cent), Export credits was US\$1,059.68 (5.31 per cent), Commercial debt US\$1,074.65 (5.38 per cent), Bilateral facilities were US\$867.39 million (4.34 per cent), and International capital market bonds were US\$1,530.51 million (7.66 per cent).

Domestic Debt Stock

The domestic debt stock stood at US\$12,569.83 million at the end of August 2013. Of this amount, 27.5 per cent was in short-term instruments (instruments with maturities of 1 year and below), 48.72 per cent in medium-term instruments, and 21.6 per cent in long-term instruments, while standard term loan (usually with commercial banks) constituted 2.18 per cent.

Risks Associated with the Debt Structure

Risk management is the process of identifying the level of risk that an entity wants to incur, measuring the level of risk that an entity currently faces, taking actions that bring the actual level of risk to the desired level and monitoring the new level of risk so that it continues to be aligned with the desired level. Process is continuous

and may require alterations in any of these activities to reflect new policies, preferences and information.

Sound debt management practices stress the importance of analysing and monitoring the risks inherent in the structure of public debt as well as mitigating them, while taking into account the costs of doing so. In the trade-off between expected costs and risks, very often cost-minimisation is given priority over risk-mitigation in public debt management. However, an excessive focus on cost savings at the expense of risk reduction can have serious consequences for an economy and potentially lead to crises.

Debt Sustainability Ratios

Ghana's solvency ratios through the year to date show that the nation's debt is sustainable with a moderate degree of debt distress. As shown in Table below, total public debt-to GDP ratio was estimated at 49.44 per cent as at end August 2013, of which total external debt/GDP was 22.11 per cent, while total domestic debt was 27.33 per cent.

DEBT TO GDP AS AT END AUGUST 2013	PERCENT (%)
PUBLIC DEBT	49.44%
EXTERNAL DEBT	22.11%
DOMESTIC DEBT	27.33%

External Debt Sustainability Analysis

The debt trajectories remain the same as in the previous DSA with two indicators breaching the thresholds under the baseline. 7 The PV of debt-to-GDP ratio would breach the threshold in 2016 though the ratio is trending down with a peak in 2015. On the other hand, debt service-to-revenue ratio is projected to hover above the threshold over the projection period with some spikes associated with Eurobond redemptions. Recently-issued Eurobonds have an amortization nature, with a somewhat smooth redemption schedule. Revenue enhancement measures supported by proactive debt management to further smooth and lengthen the maturity profile would improve this indicator. 7. The debt outlook remains sensitive to standard shocks to key macro variables. Under the standard stress tests, the relevant debt indicators are worst and the most extreme over the first 10-year projection period to the exchange rate and exports shocks, leading to breaches of other indicators against the thresholds in addition to the debt service-to-revenue ratio.

Public Debt Sustainability Analysis

Strong fiscal adjustments and an adequate financing package should lower Ghana's total public debt (Table 4, Figure 3). Though PV of debt-to-GDP ratio would breach the public debt benchmark in the first few years,

it is expected to decline to around 40 percent of GDP by the end of the projection period with fiscal reforms on track. The other debt indicators are also expected to improve and stabilize. Though all indicators show sustainable paths under the baseline scenario, they could be on an explosive path under the historical and the most extreme shock scenario (with abrupt real exchange rate depreciation).

Contingent liabilities emanating from state-owned enterprises (SOEs) and NPLs of the banking sector associated with these entities are concerns. There had been legacy issues in SOEs, including inefficient management, limited revenue generating capacity partly due to low levels of administered tariff setting, and cross arrears with the government and other SoEs. The authorities have recently raised tariffs and levies on petroleum products to address SOEs imbalances and plan on comprehensively clearing their arrears and restructuring their unsustainable debt. They also plan to improve the SOE governance and the government oversight with support of the World Bank. Although in aggregate, the banking system is profitable and has adequate levels of capital and liquidity, a recently conducted Asset Quality Review revealed weaknesses in loan classification and provisioning practices among some banks. The BoG has designed with IMF staff, a roadmap for enhancing the resilience of the banking sector and will require banks to re-classify loans to SOEs and book additional provisions if necessary. The comprehensive reform program for SOEs should also help to enhance the soundness of the banking sector. The staff will continue to monitor these vulnerabilities and contingent liabilities to avoid them jeopardizing debt sustainability.

CONCLUSION

The main contribution of this paper is to propose a risk based measure to assess debt sustainability. The main insight that lead to this measure is that debt in emerging market economies is quite risky. An increasingly larger number of models and body of empirical evidence show that emerging market economies lack the natural stabilizing features that allow and make countercyclical policies effective. For example, during a recession, real interest rates tend to fall in developed economies, which makes room for larger primary expenditures given the reduction in interest payments. In emerging market economies, often the reverse happens: real interest rate rises, government revenues fall and government outlays increase, thereby negatively leveraging the debt impact. The proposed measure is constructed from a framework that combines a statistical model to uncover the stochastic relations among the variables that directly or indirectly influence the debt accumulation with a

simulation engine that computes future paths for the debt to GDP ratios. The Monte Carlo simulations allow us to compute “risk probabilities”, i.e., probabilities that the simulated Debt to GDP ratio exceeds a given threshold deemed risky (say, 75% of GDP). The time-series of such probabilities is then used to investigate whether or not it is correlated with the market risk assessment, measured by the spread on sovereign dollar denominated debt.

In this paper we study the question of debt sustainability from a risk management perspective. The debt accumulation equation for any country involves variables that are stochastic and closely intertwined. When these aspects are taken into consideration the notion of debt sustainability is expanded to studying the stochastic properties of the debt dynamics. We illustrate the methodology by studying the Brazilian and Ghanaian case. We find that even though the debt could be sustainable in the absence of risk, there are paths in which it is clearly unsustainable. Furthermore, we show that properties of the debt dynamics are closely related to the spreads on sovereign dollar denominated debt.

The application of our methodology for Brazil shows that even though the debt could be sustainable in the absence of risk, there are many paths in which it is clearly unsustainable. I.e., although “on average” the Brazilian debt is sustainable, the correlation structure under which the Brazilian economy operates poses a huge question mark on the debt sustainability.

REFERENCE

Prepared in collaboration with Ghanaian authorities. The previous DSA was prepared in January 2016 (IMF Country Report No. 16/16).

The World Bank’s Country Policy and Institutions Assessment (CPIA) ranks Ghana as a medium performer in terms of the quality of policy and institutions (the average CPIA in 2012–14 is 3.61). Thus, the external debt burden thresholds for Ghana are (i) PV of debt-to-GDP ratio: 40 percent; (ii) PV of debt-to-exports ratio: 150 percent; (iii) PV of debt-to-revenue ratio: 250 percent; (iv) debt service-to-exports ratio: 20 percent; and (v) debt service-to-revenue ratio: 20 percent. The 2014 CPIA has been released in July 2015 by the World Bank, and Ghana’s score has deteriorated to 3.37, which contributed to the downgrading of Ghana to a medium performer from a high performer.

IMF, World Economic Outlook, September 2003.

Missale, A., F. Giavazzi and P. Benigno. “How is Debt Managed: Learning from Fiscal Stabilizations”, RePEc:igi:igierp:174, September 2000.

Reinhart, C., K. Rogoff and M. Savastano. “Debt Intolerance,” *Brookings Papers on Economic Activity*, 2003. Rothenberg, T. and J. Stock. “Inference in a Nearly Integrated Autoregressive Model with Nonnormal Innovations,” *Journal of Econometrics*, 80 (1997), 269-286

Garcia, M. and M. Valpassos,, “Capital Flows, Capital Controls and Currency Crisis: The Case of Brazil in the Nineties” in Felipe Larrain, ed., *Capital Flows, Capital Controls, and Currency Crises: Latin America in the 1990s*, University of Michigan Press, 2000.

Box 3.1 (Data on Public Debt in Emerging Market Economies) of the last World Economic Outlook (IMF 2003) describes the many difficulties involved in gathering such data.

The Brazilian Central Bank web page (www.bcb.gov.br).

The Ghanaian Central Bank web page , www.bod.gov.gh